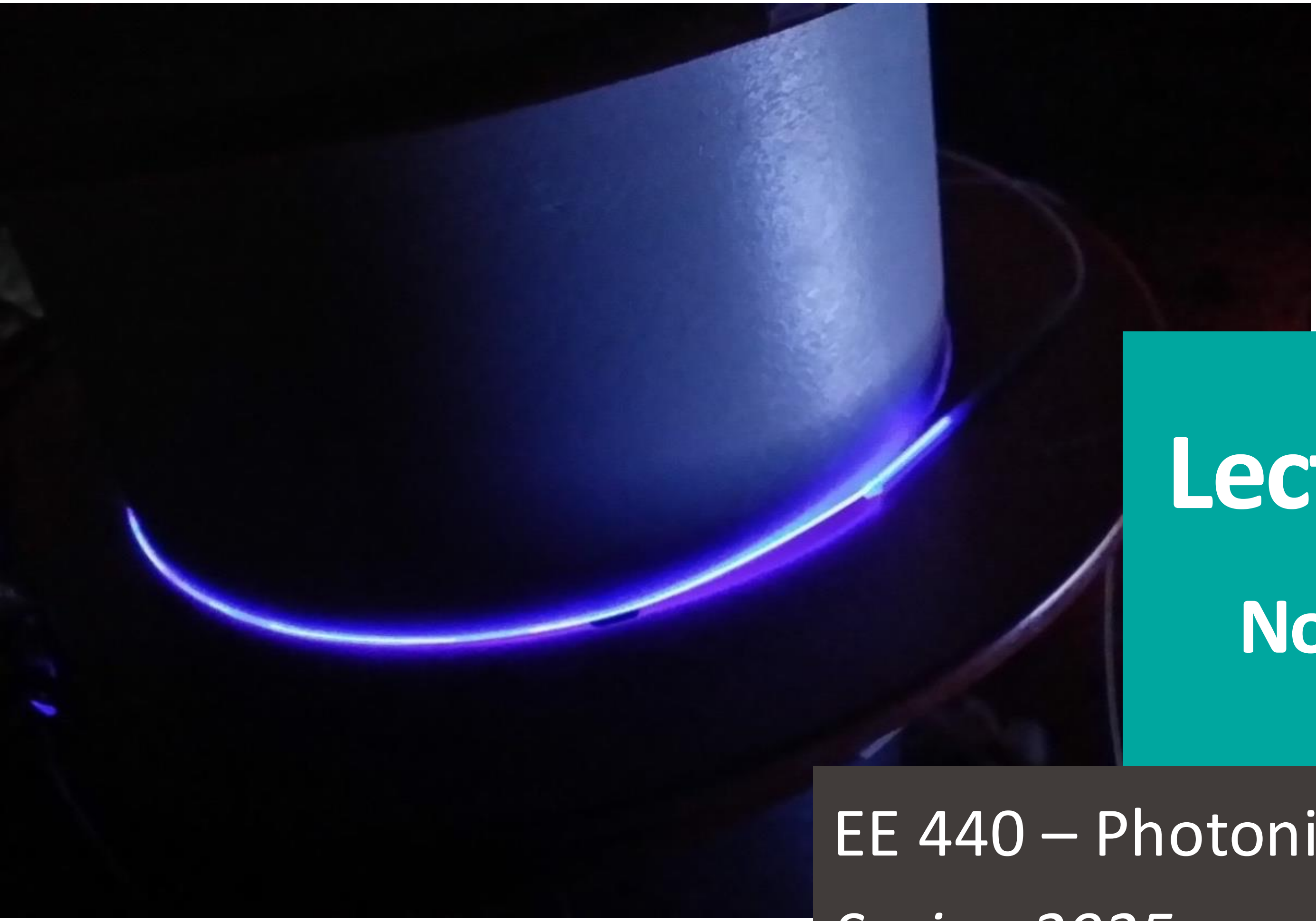


Lecture 7

Nonlinear effects

EE 440 – Photonic systems and technology

Spring 2025

Ecole
Polytechnique
Fédérale de
Lausanne

Prof. Camille Brès - camille.bres@epfl.ch

Lecture 7 outline

Introduction to nonlinear phenomena

- Why and when are they important
- General concepts

Different types of fiber nonlinearities

- Kerr effects
- Solitons
- Scattering effects

Introduction to nonlinear optics

Why study fiber nonlinearities?

What transmitted power would you choose in a fiber optic link ?

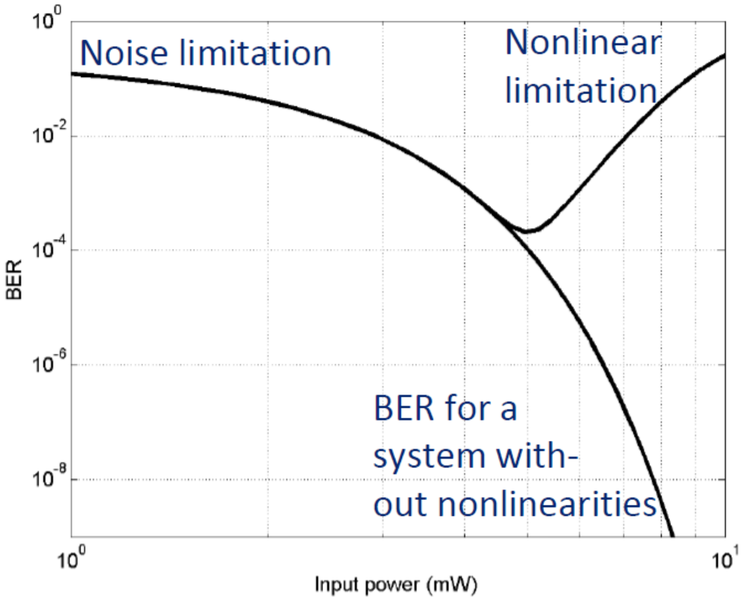
- Lasers can have high output powers.
- Signal to noise ratio (SNR) is proportional to input power .
- Clearly higher input power is always better (?)

No, actually it is not ….

The nonlinear trade off:

- At low power, systems are limited by ***noise***
- At high power, systems are limited by ***nonlinearities***

There exist an optimum launch power



EE – 440 | C.-S. Brès

Nonlinear effects

When is a phenomenon "nonlinear"?

- When superposition does not apply.
- When it cannot be explained by a mathematical relationship of proportionality.

In nonlinear optics:

- Light cannot be viewed as a superposition of independently propagating spectral components.
- Spectral components interact.
- New frequencies can be generated, existing components can lose power.
- It is possible to control light with light.

Fiber nonlinearities

Under an applied optical field $\mathbf{E}$ a dielectric becomes polarized ($\mathbf{P}$) due to its susceptibility χ (i.e. generation of dipoles)

- Linear optics (i.e. optics of 'weak' light):

$$\mathbf{P} = \varepsilon_0 \chi^{(1)} \mathbf{E}$$

ε_0: vacuum permittivity
$\chi^{(n)}$: n^{th} order of susceptibility

The response becomes nonlinear under intense electromagnetic fields.

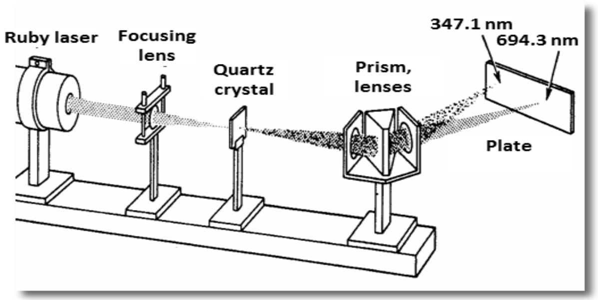
- Nonlinear optics (i.e. optics of 'intense' light):

$$\mathbf{P} = \varepsilon_0 \chi^{(1)} \mathbf{E} + \varepsilon_0 \chi^{(2)} \mathbf{E}^2 + \varepsilon_0 \chi^{(3)} \mathbf{E}^3 = \mathbf{P}_L + \mathbf{P}_{NL}$$

$\chi^{(2)}$ effects

$\chi^{(2)}$ is responsible, among others, for many crucial frequency mixing effects, for example:

- The electro optic effect (mixing between optical and RF frequencies)
- Second harmonic generation (SHG): $\omega_1 + \omega_1 = 2\omega_1$



$$\omega_1 + \omega_1 = 2\omega_1$$

$$1064\, nm \rightarrow 532\, nm$$

- $\chi^{(2)} = 0$ for centro-symmetric or amorphous materials (such as Si, SiO_2, SiN etc…)

$\chi^{(3)}$ effects

Lowest order nonlinear effects in optical fibers originate from 3rd order susceptibility $\chi^{(3)}$.

$$\mathbf{P} = \varepsilon_0 \chi^{(1)} \mathbf{E} + \varepsilon_0 \chi^{(3)} \mathbf{E}^3$$

Linear refractive index

We can show that the medium linear refractive index n and power absorption coefficient α are related to 1st order electric susceptibility $\chi^{(1)}$

Start from the relative electric permittivity. By definition:

$$\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P} = \varepsilon_0 \left(1 + \chi^{(1)}\right) \mathbf{E} \equiv \varepsilon_0 \varepsilon_r \mathbf{E}$$

$$\varepsilon_r = \left(1 + \chi^{(1)}\right)$$

ε: complex electric permittivity
ε_r :complex dielectric constant

Let $\underline{n}$ be the complex refractive index: $\underline{n} = \frac{c_0}{c} = \sqrt{\varepsilon_r \mu_r} \approx \sqrt{\varepsilon_r}$

$$\underline{n} = \sqrt{1 + \chi^{(1)}}$$

Linear refractive index

The complex propagation constant is therefore:

$$\beta_p(\omega) = \beta_L(\omega) + i\frac{\alpha(\omega)}{2} = k_0\underline{n}$$

$$k_0\, n(\omega) + i\frac{\alpha(\omega)}{2} = k_0\sqrt{1+\chi^{(1)}}$$

$$n(\omega) + i\frac{\alpha(\omega)}{2k_0} = \sqrt{1 + \mathrm{Re}(\chi^{(1)}) + i\mathrm{Im}(\chi^{(1)})}$$

Linear refractive index in weakly absorbing media

In a weakly absorbing media: $\mathrm{Im}\left(\chi^{(1)}\right) \ll 1 + \mathrm{Re}\left(\chi^{(1)}\right)$

We can show that :

$$n(\omega) \approx \sqrt{1 + \mathrm{Re}(\chi^{(1)})}$$

$$\alpha(\omega) \approx \frac{k_0}{n} \mathrm{Im}\left(\chi^{(1)}\right) = \frac{\omega}{n c_0} \mathrm{Im}\left(\chi^{(1)}\right)$$

Nonlinear refractive index n_2

Most nonlinear effects in optical fibers originate from nonlinear refraction.

- Refers to the intensity dependence of the refractive index
- Characterized by the nonlinear refractive index n_2

To solve for n_2, we start by expressing the nonlinear polarization. We can show that:

$$\mathbf{P}_{NL} \approx \varepsilon_0 \chi^{(3)} \frac{3}{4} |E|^2 \mathbf{E}$$

Considering the nonlinear polarization as a perturbation :

$$\mathbf{P} = \varepsilon_0 \chi^{(1)} \mathbf{E} + \mathbf{P}_{NL} \approx \varepsilon_0 \chi^{(1)} \mathbf{E} + \varepsilon_0 \Delta\chi \, \mathbf{E}$$

- With $\Delta\chi = \chi^{(3)} \frac{3}{4} |E|^2$

Nonlinear refractive index n_2

The variation in susceptibility $\Delta\chi$ results in a refractive index variation Δn

$$\Delta n = \frac{3}{8n}\chi^{(3)}|E|^2$$

It is more common to use the intensity I instead of the field: $I = \frac{c_0 \varepsilon_0 n}{2}|E|^2$

We get: $\Delta n = \frac{3}{4c_0\varepsilon_0 n^2}\chi^{(3)} I \equiv n_2 I$

The intensity nonlinear refractive index is (in cm^2/W): $n_2 = \frac{3}{4c_0\varepsilon_0 n^2}\chi^{(3)}$

The refractive index become *dependent on light intensity:* **optical Kerr effect**

$$n(\omega, I) = n(\omega) + n_2 I$$

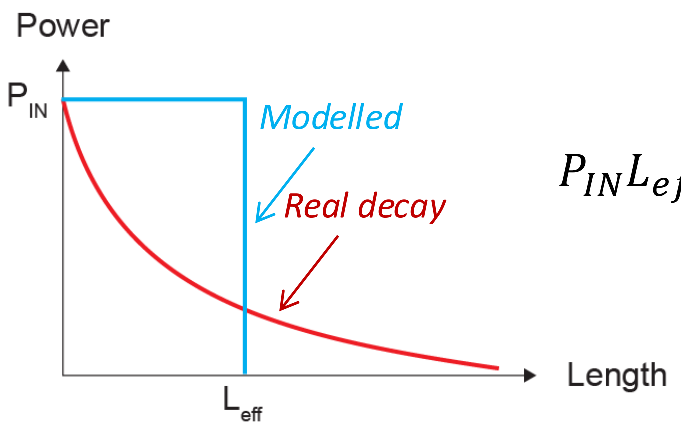
Effective length and area

Nonlinear effects depend on intensity (power).

- As signal propagates, power decreases due to attenuation

$$P(z) = P_{IN}\exp(-\alpha z)$$

- Use a model that considers constant power over an *effective length*

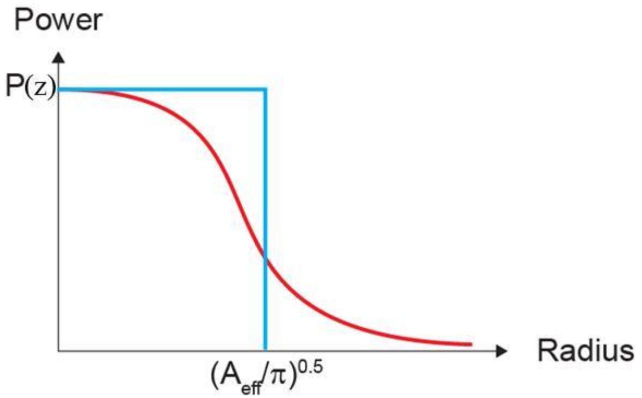


$$P_{IN}L_{eff} = \int_0^L P(z)dz$$

$$L_{eff} = \frac{1-e^{-\alpha L}}{\alpha}$$

At given power, intensity is inversely proportional to optical mode area.

- Power is not uniformly distributed within the fiber cross section
- Model with effective cross sectional area A_{eff}
- Effective intensity of a pulse is $I = \frac{P(z)}{A_{eff}}$



➢ For single mode operation:

$$A_{eff} \approx \pi r_{core}^2$$

The optical Kerr effect

The refractive index is intensity dependent: $n(\omega, I) = n(\omega) + n_2 I$

The Kerr effect gives rise to nonlinear phenomena including:

- Self-phase modulation (SPM)
  - Causes spectral broadening
- Cross-phase modulation (XPM)
  - Causes frequency shifts between channels of different frequencies
- Four-wave mixing (FWM)
  - Causes power exchange between channels of different frequencies

Self phase modulation

Self phase modulation (SPM)

As a result of the Kerr effect, a wave can act upon itself and experience an *intensity related phase shift:*

$$\varphi(z) = \beta z = \frac{2\pi}{\lambda_0} n(I) z$$

$$\varphi(z) = \frac{2\pi}{\lambda_0} (n + n_2 I\,) z$$

$$\varphi(z) = \varphi_0 + \Delta\varphi(z) \quad \text{with} \quad \Delta\varphi(z) = \frac{2\pi}{\lambda_0} n_2 I z$$

If light intensity is time dependent $I(t)$, the extra phase shift $\Delta\varphi(z)$ also becomes time-dependent: *signal is phase modulated by its own intensity*.

This is called self phase modulation

Self phase modulation

$$\Delta\varphi(z) = \frac{2\pi}{\lambda_0} n_2 I z \quad \Longrightarrow \quad \Delta\varphi(z) = \frac{2\pi}{\lambda_0} n_2 \frac{P(z)}{A_{eff}} z$$

$$\Delta\varphi(z) = \gamma P(z) z$$

$$\gamma = \frac{2\pi n_2}{\lambda_0 A_{eff}}$$

Additional phase shift φ_{NL} caused by self phase modulation after distance L :

$$\varphi_{NL} = \int_0^L \Delta\varphi(z) dz = \gamma \int_0^L P(z) dz$$

$$\varphi_{NL} = \gamma P_{IN} L_{eff}$$

Nonlinear phase time varying input

Assume the input signal has a time varying envelope:

$$A(0,t) = \sqrt{P_0}U(0,t)$$

Peak power P_0

Normalized envelope

- The input power is given by: $P_{IN}(t) = |A(0,t)|^2 = P_0|U(0,t)|^2$
- The nonlinear phase shift: $\varphi_{NL}(t) = \gamma P_{IN}(t)L_{eff} = \gamma P_0|U(0,t)|^2 L_{eff}$
- The maximum phase shift φ_{NL}(max) is therefore when $|U(0,t)|^2 = 1$

$$\varphi_{NL}(t) = \gamma P_0|U(0,t)|^2 L_{eff}$$

$$\varphi_{NL}^{max} = \gamma P_0 L_{eff}$$

Induced nonlinear Chirp

<u>Chirp:</u> $\delta\omega(t) = -\frac{\partial\varphi}{\partial t}$ and $\varphi_{NL}(t) = \gamma P_{IN}(t) L_{eff}$

<u>For a Gaussian:</u>
(initially unchirped)

$$A(t,0) = \sqrt{P_0}\exp\left[-\frac{1}{2}\left(\frac{t}{T_0}\right)^2\right]$$

$$P_{IN}(t) = |A(0,t)|^2 = P_0 \exp\left[-\left(\frac{t}{T_0}\right)^2\right]$$

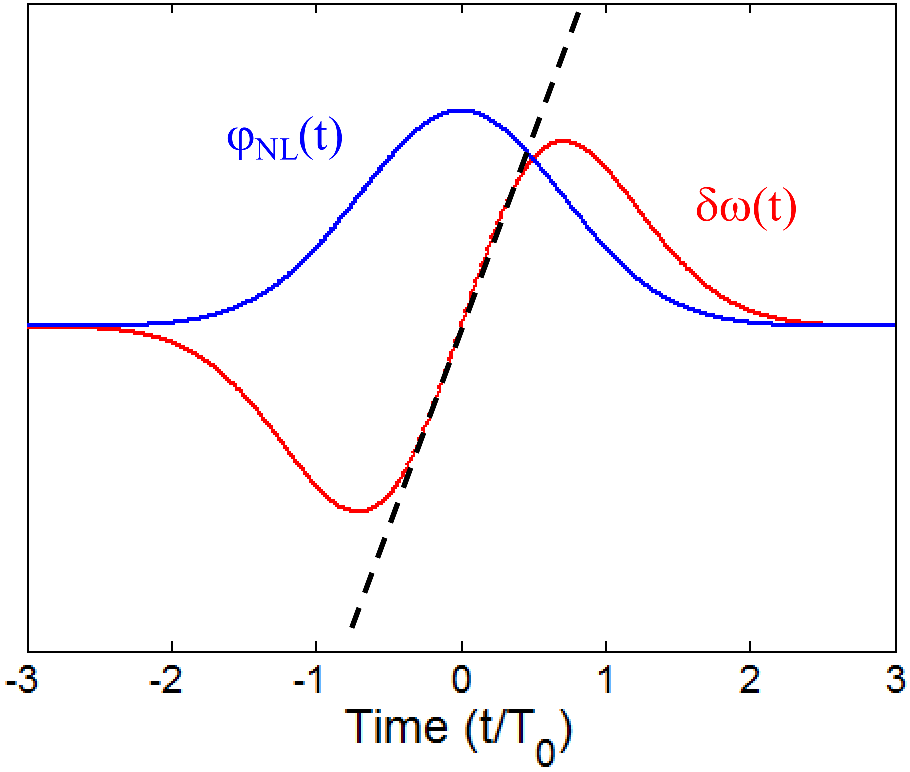
$$\varphi_{NL}(t) = \gamma P_0 \exp\left[-\left(\frac{t}{T_0}\right)^2\right] L_{eff}$$

Nonlinear phase shift

$$\delta\omega(t) = 2\gamma P_0 L_{eff} \frac{t}{T_0^2} \exp\left[-\left(\frac{t}{T_0}\right)^2\right]$$

Induced nonlinear chirp

SPM impact on pulses



Nonlinear phase shift φ_{NL} is directly proportional to $|A(0,t)|^2$

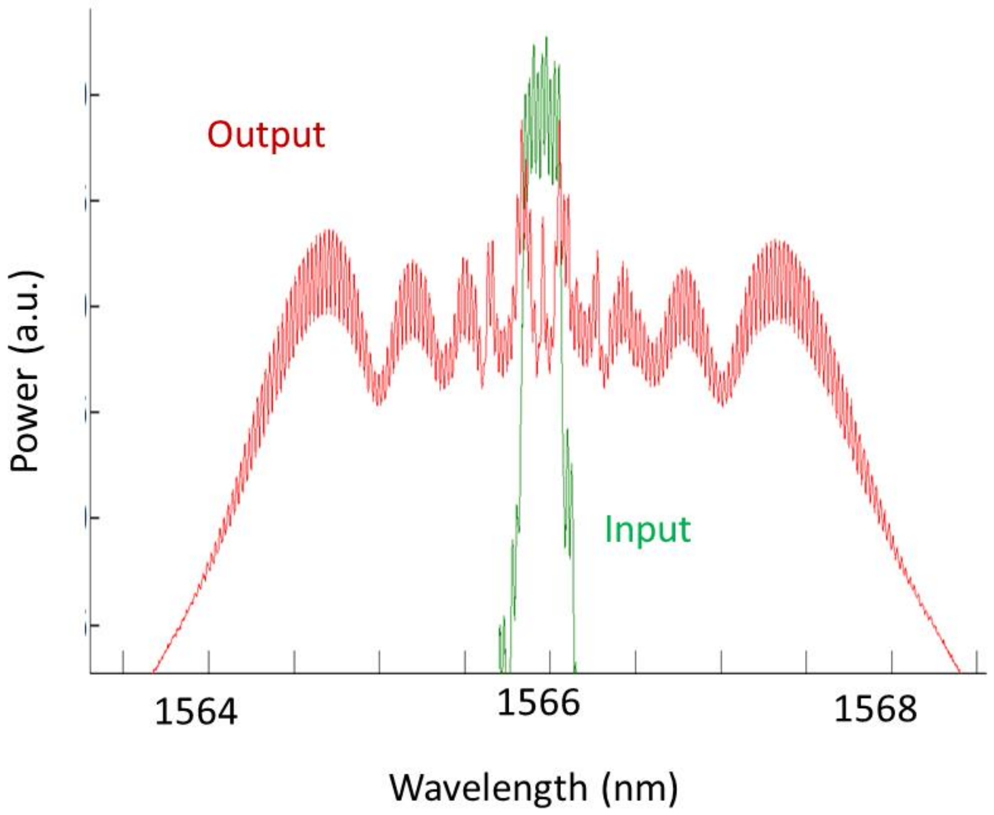
- Temporal variation identical to pulse shape

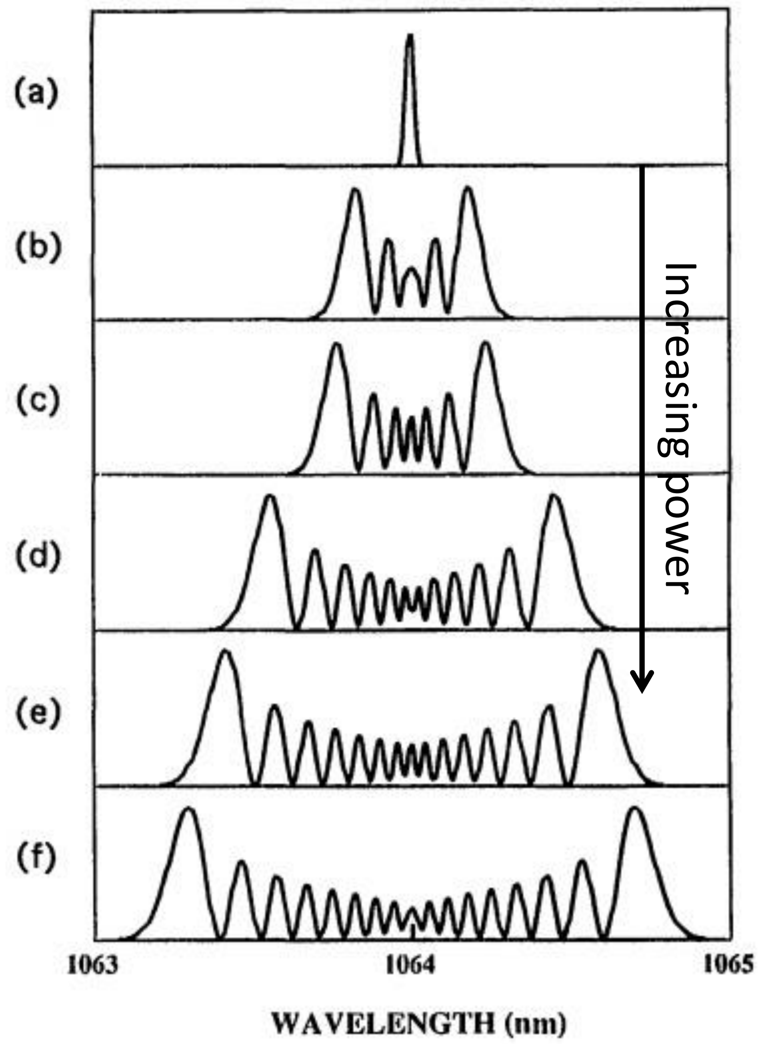
Induced chirp is negative near the leading edge and becomes positive

- Linear up chirp over a large central region

- In the absence of dispersion , the pulse shape will not change
- SPM induces a chirp and continually broadens the spectrum
- The chirping depends on the pulse shape

SPM impact on spectrum

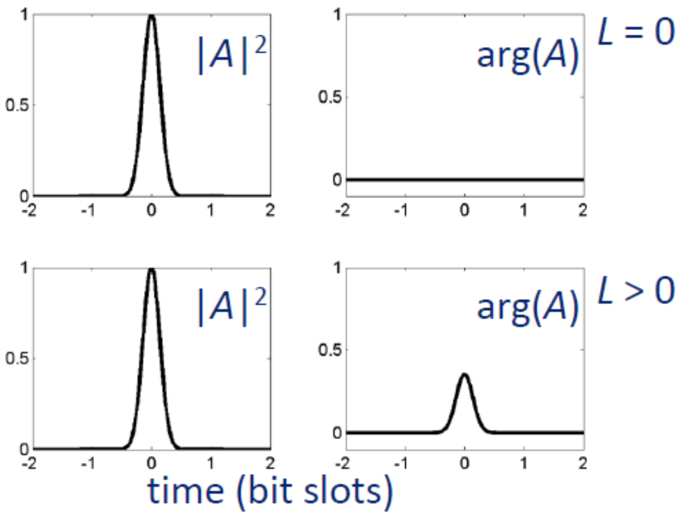


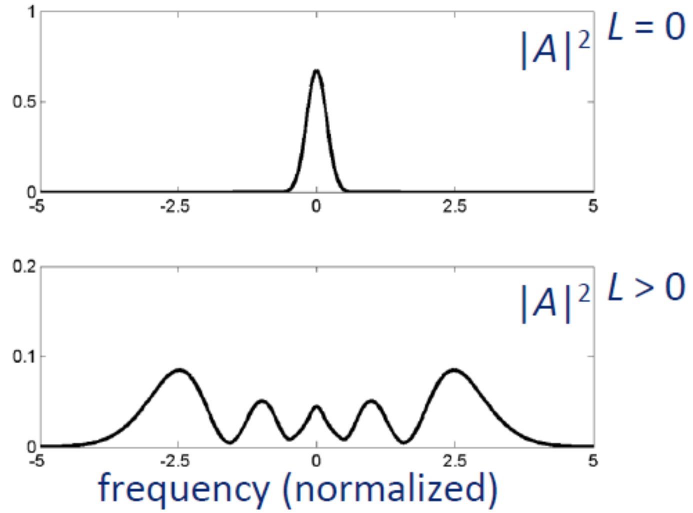


Nonlinear propagation and soliton formation

Nonlinear propagation

Let's assume purely <u>nonlinear medium</u>, i.e. $dn/d\omega = 0$, $n_2 \neq 0$





In the time domain

- Pulse is phase modulated by its own intensity.
- No effect on distribution: pulse shape is unchanged
- Pulse phase shift is introduced (SPM)

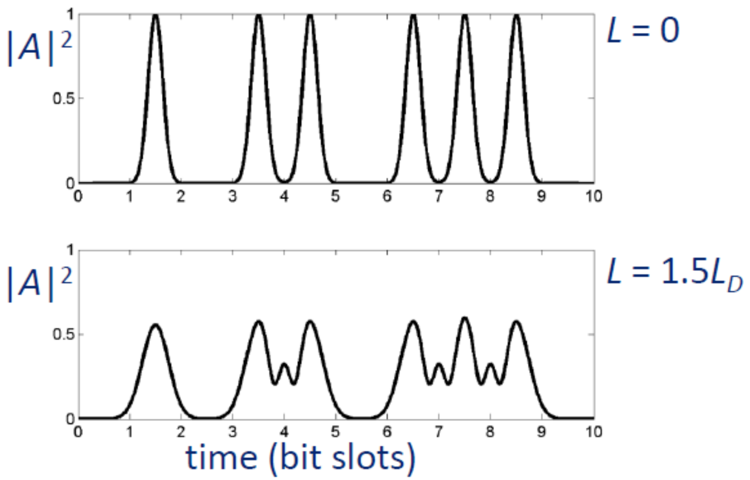
In the frequency domain

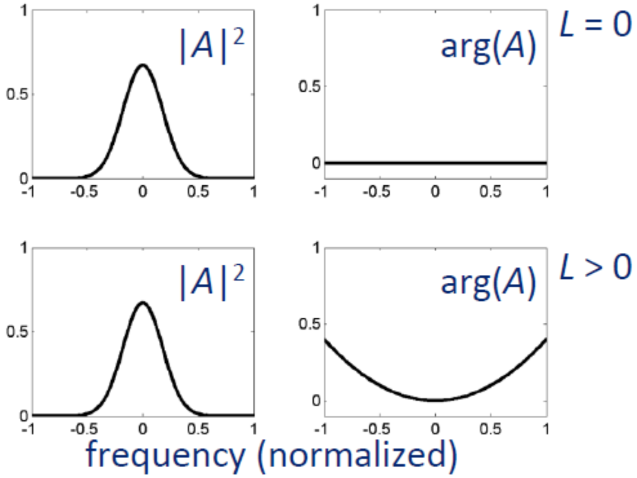
- The spectrum is broadened due to the phase modulation
- Energy is conserved

The length scale for nonlinearity is the nonlinear length $L_{NL} = 1/\gamma P_0$

Linear dispersive propagation

Let's assume purely dispersive medium, i.e. $n(\omega)$ and $n_2 = 0$





In the time domain

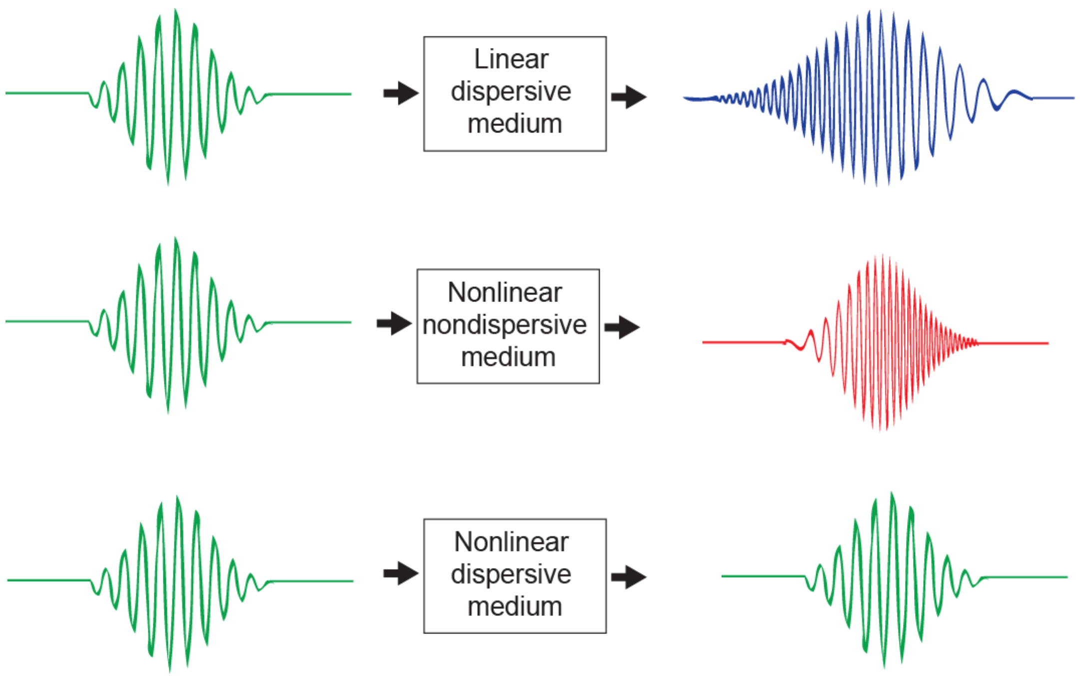
- Pulse experiences temporal spreading.
- A phase shift (chirp) will become an amplitude change

In the frequency domain

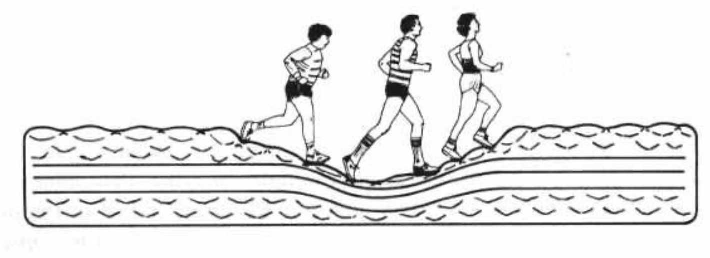
- Amplitude is not changed
- The phase is modulated

The length scale for dispersion is the dispersive length $L_D = T_0^2/|\beta_2|$

Propagation in dispersive + nonlinear medium



Analogy…



Runners create a moving valley that pulls slower runners and retards faster ones. (otherwise, the faster runners would move ahead and the slower behind)

It is possible to demonstrate that for such a medium a pulse exist which will propagate *without its envelope being modified*. Moreover it is a stable solution.

Solitons

The Scottish engineer John Scott Russell (1808–1882) recalls the first observation of a soliton, while horseback riding near a canal:

”…rolled forward with great velocity, assuming the form of a large solitary elevation, a rounded, smooth and well-defined heap of water, which continued its course along the channel apparently without change of form or diminution of speed…

… and after a chase of one or two miles I lost it in the windings of the channel. Such, in the month of August 1834, was my first chance interview with that singular and beautiful phenomenon which I have called the Wave of Translation”

GVD and SPM – going back to the NLSE

Assumptions:

- Lossless fiber, far from the zero dispersion wavelength (β_2 dominates).
- Gives very few explicit solutions.

$$\frac{\partial A}{\partial z} + i\frac{\beta_2}{2}\frac{\partial^2 A}{\partial t^2} - i\gamma|A|^2 A = 0$$

Solitons are special solutions which form stable pulses.

- They propagate undistorted or in periodic manner (depend on the soliton 'order')
- Bright solitons are obtained when $\gamma\beta_2 < 0$ (i.e. always in the anomalous dispersion region of a silica fiber since $\gamma > 0$).
- Dark solitons exist for normal dispersion.

Important parameters for solitons

The formation of solitons critically depends on:

- The fiber parameters (dispersion and nonlinearity).
- The input pulse parameters (width and peak power).
- We define a soliton order as N:

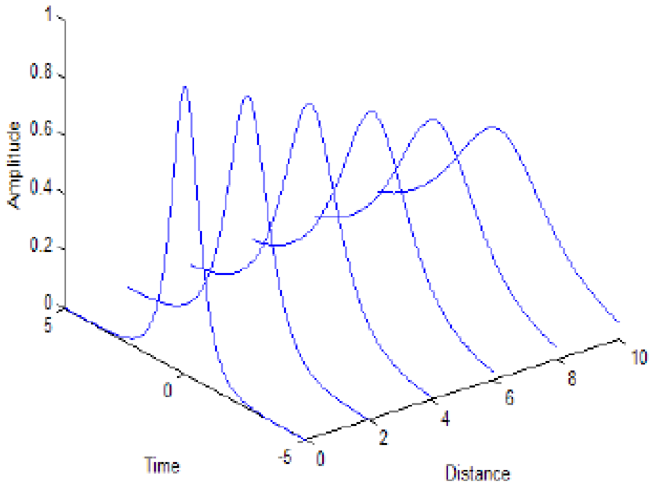
$$N^2 = \gamma P_0 L_D = \gamma P_0 \frac{T_0^2}{|\beta_2|}$$

$$A(z,t) = \sqrt{P_0}\,\underbrace{\operatorname{sech}\left(\frac{t}{T_0}\right)}_{\text{Envelope shape}}\,\underbrace{\exp\left(i\frac{z}{2L_D}\right)}_{\text{Phase factor}}$$

Envelope shape *Phase factor*

- Effects of dispersion are exactly compensated by nonlinearities
- The pulse amplitude does not depend on ‘z’ and remains unchanged.
- The input pulse acquired a phase shift upon propagation.

Soliton pulse propagation

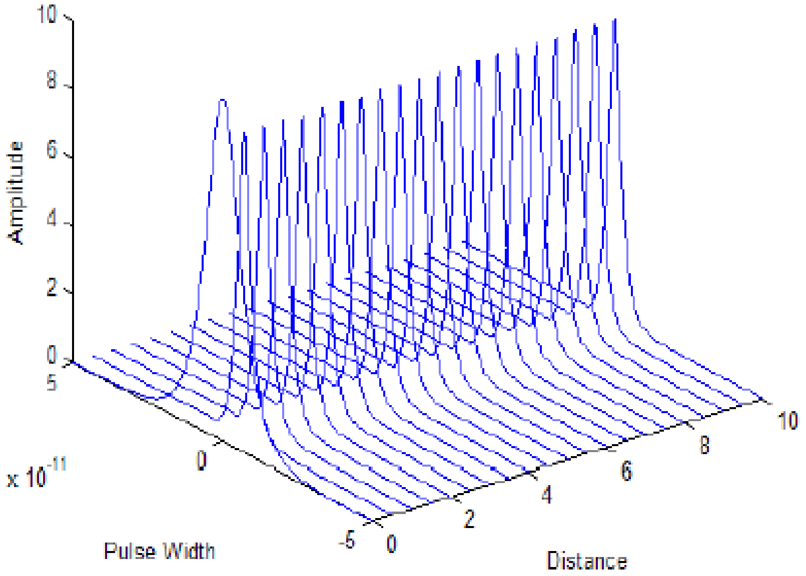


Power below soliton threshold:

- Initial pulse broadens as it propagates.
- Propagation dominated by ordinary GVD.

Above soliton power threshold:

- Initial Gaussian pulse for which N =1
- Pulse adjusts its shape, and width
- Attains a sech profile for $z >> L_D$



Problems with Soliton communication

Soliton transmission is typically not used and nonlinear effects are kept low by design

Soliton balance condition depends on the power.

- Fibers are lossy and the balance will be disturbed

Solitons interact.

- There is no superposition anymore
- Soliton pulses can attract of repel each other

How to do amplitude modulation ?

Multi-wavelength effects

When more than one wavelength are co-propagating, we can often consider these as separate waves.

- They interact via cross-phase modulation (XPM) and four-wave mixing (FWM).

Consider the case of 2 λs: $A = a\exp(-i\omega_a t) + b\exp(-i\omega_b t)$

- Nonlinearities manifest through the $|A|^2 A$ term in the NLSE

$$\underbrace{|a|^2 a e^{-i\omega_a t}}_{\text{SPM}} + \underbrace{|b|^2 b e^{-i\omega_b t}}_{\text{SPM}} + \underbrace{2|b|^2 a e^{-i\omega_a t}}_{\text{XPM}} + \underbrace{2|a|^2 b e^{-i\omega_b t}}_{\text{XPM}} + \underbrace{a^* b^2 e^{-i(2\omega_b-\omega_a)t}}_{\text{FWM}} + \underbrace{b^* a e^{-i(2\omega_a-\omega_b)t}}_{\text{FWM}}$$

Intensity of the wave influences its own phase

Wave 1 intensity influences phase of wave 2

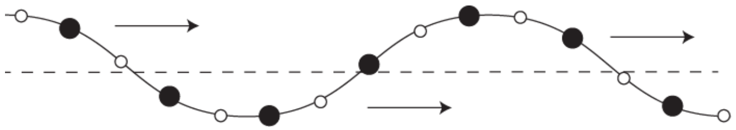
Waves at new oscillating frequencies appear: $2\omega_b - \omega_a$ and $2\omega_a - \omega_b$

- XPM/FWM depend on temporal overlap/phase matching: dispersion decreases their efficiencies.

Scattering processes

Stimulated scattering

In a dense medium made of polyatomic molecules, cohesive force between molecules allows them to vibrate collectively.



Oscillatory movement of the entire molecular chain

- Classical wave, slow vibration
- Acoustic-like vibration
- ***Brillouin scattering***

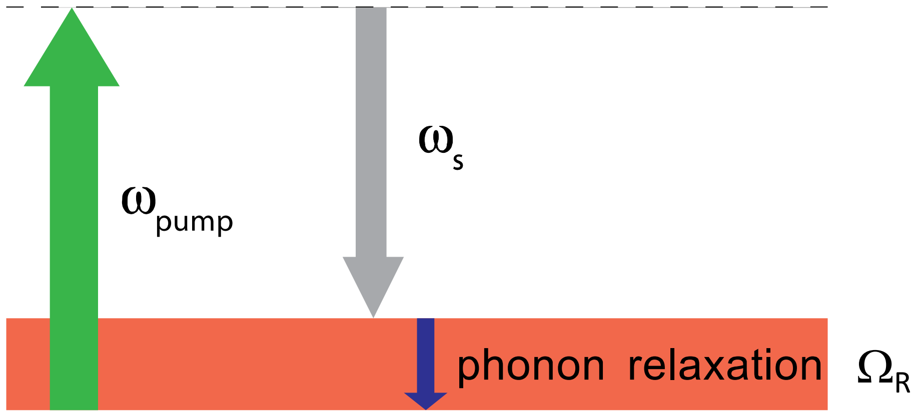
Oscillatory movement inside the molecular chain

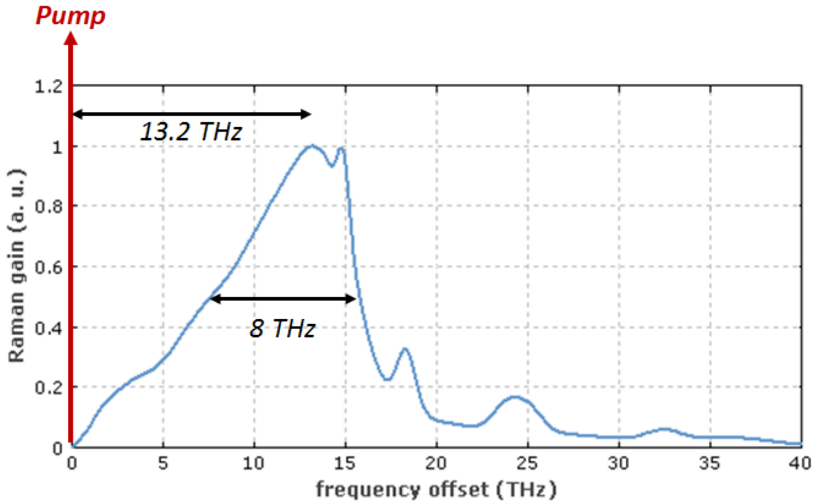
- Quasi standing wave, fast vibrations
- Optical-like vibration
- ***Raman scattering***

Raman scattering

It is the scattering of light from vibrating silica molecules.

- Strong input light at ω_{pump} can excite vibrations of the molecules: energy is therefore transferred to the lattice (optical phonons).
- To satisfy conservation of energy, photons at ω_s with reduced energy are generated.
- The vibration frequency $\Omega_R = \omega_{pump} - \omega_s$ depends on the material.
- Amorphous nature of silica results in broad possible band of vibrational states





Raman threshold

When another wave at frequency ω_s (called Stokes beam) is present together with the pump at ω_{pump} , it will be amplified.

Even in the absence of an input, this Stokes beam can build up.

- Requires that the pump power is large enough
- Spontaneous Raman scattering from noise acts as seed for the process.

Raman threshold is defined as the input pump power (P_P) at with output pump and Stokes power (P_S) becomes equal at the fiber output.

The Raman threshold is reached for

$$P_{th} \approx \frac{16A_{eff}}{g_r L_{eff}}$$

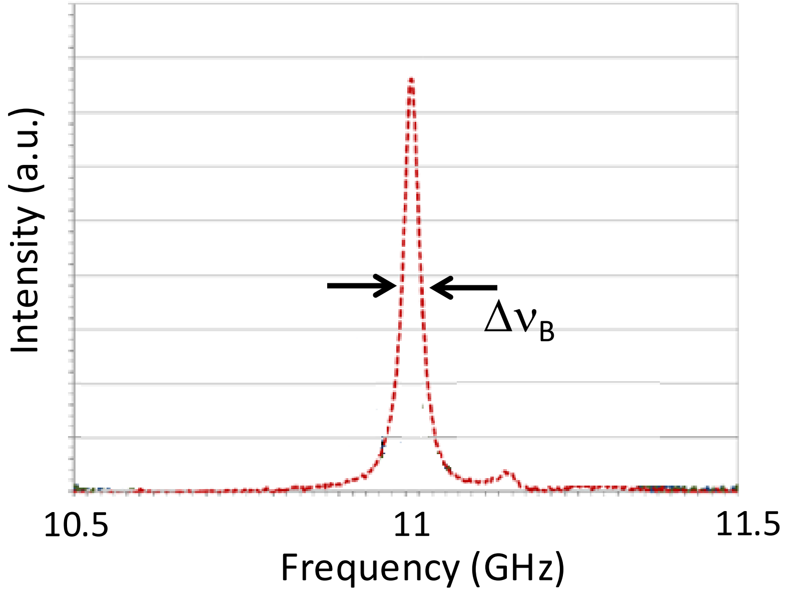
With g_r the Raman gain coefficient

Stimulated Brillouin scattering (SBS)

SBS can be described as a nonlinear interaction between a pump and Stokes beam through an acoustic wave

- Leads to backscattered light, down shifted in frequency from the pump.
- Quantum mechanically: pump photon at ω_p is annihilated to create Stokes photon at ω_s and acoustic phonon $\Omega_B = \omega_p - \omega_s$
- This acoustic phonon has a much lower frequency than the optical phonon.
- Brillouin frequency shift depends on the acoustic velocity in the material

$$\nu_B = \frac{\Omega_B}{2\pi} = 2n_p \frac{v_A}{\lambda_p}$$



Brillouin threshold

Similar to Raman, when another wave at frequency ω_s is present together with the pump, it will be amplified.

Even in the absence of an input, the Stokes beam can build up:

- Requires that the pump power is large enough
- Spontaneous Brillouin scattering acts as the seed for the build up

Brillouin threshold is defined as for the Raman threshold.

The Brillouin threshold is reached for

$$P_{th} \approx 21 \frac{1.5 A_{eff}}{g_B L_{eff}} \left(1 + \frac{\Delta\nu_p}{\Delta\nu_B}\right)$$

With g_B the Brillouin gain coefficient
$\Delta\nu_P$ the pump laser linewidth
$\Delta\nu_B$ the Brillouin gain bandwidth

SBS in systems

SBS limits the continuous wave power that can be injected in a fiber:

- For narrow $\Delta\nu_P$ the threshold for SBS is low (10's of mW).
- Pump power is backscattered.

The phase matching condition necessary for SBS is strongly dependent on the acoustic velocity ν_A in the medium:

- Varies with respect to parameters such as temperature or density.
- Density can be varied by strain.
- Change in accoustic velocity leads to a measurable shift in ν_B.

Brillouin scattering can be used to create very efficient distributed sensors for temperature or strain

- Entire fiber is sensing element